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12MMD/MDE/MCM/MEA/MAR/MST11

First Semester M.Tech. Degree Examination, Dec.2013/Jan.2014
Applied Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Define with suitable examples
   i) Inherent error.
   ii) Significant figures.
   iii) Truncation error.
   iv) True percentage relative error. **(10 Marks)**

b. A parachutist of mass 68.1kgs jumps out of a stationary hot air balloon, use $\frac{dv}{dt} = g - \frac{c}{m}v$ to compute velocity v prior to opening the chute. The drag coefficient is equal to 12.5kg/s. Given that g = 9.8. Use analytical method to compute velocity prior to opening the chute, calculate the terminal velocity also. (Take a step size of 2 secs for computation). **(10 Marks)**

2 a. Explain bisection method. Use regular Falsi method to find a third approximation of the root for the equation tanx + tanhx = 0, which lies between 2 & 3. **(10 Marks)**

b. Explain modified Newton-Raphson method. Use modified Newton-Raphson method to find a root of the equation $x^4 - 11x + 8 = 0$ correct to four decimal places. Given $x_0 = 2$. **(10 Marks)**

3 a. Use Muller's method with guesses of x_0, x_1 and x_2 = 4.5, 5.5, 5 to determine a root of the equation $f(x) = x^3 - 13x - 12$. Perform two iterations. **(10 Marks)**

b. Find all the roots of the polynomial $x^3 - 6x^2 + 11x - 6$ using the Graffe's Root square method by squaring thrice. **(10 Marks)**

4 a. Obtain a suitable Newton's interpolation formula to find the first derivative. Use it to evaluate at x = 1.2 from the following table: **(10 Marks)**

x:	1.0	1.5	2.0	2.5	3.0
y:	27	106.75	324.0	783.75	1621.00

b. Apply Romberg's integration method to evaluate $\int_0^{1.2} \frac{dx}{1+x}$ taking stepsize h = 0.6, 0.3, 0.15. **(10 Marks)**

5 a. Solve the following set of equations by Crout's method:
$2x + y + 4z = 12$
$8x - 3y + 2z = 20$
$4x + 11y - z = 33.$ **(10 Marks)**

b. Determine the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$ by using the Partition method. **(10 Marks)**

6 a. Find all the eigen values and eigen vectors of the matrix by

$$A = \begin{bmatrix} 1 & \sqrt{2} & 4 \\ \sqrt{2} & 3 & \sqrt{2} \\ 4 & \sqrt{2} & 1 \end{bmatrix}$$

by Jacobi's method. Perform two iterations. **(10 Marks)**

b. Find the smallest eigen value in magnitude of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

using four iterations of the inverse power method. **(10 Marks)**

7 a. Define a linear transformation T: $\mathbb{R}^2 \to \mathbb{R}^2$ by

$$T(x) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix}.$$

Find the images under T of $u = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $u + v = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$. **(07 Marks)**

b. Let T : $\mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation and let A be the standard matrix. Then prove that: T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ iff the columns of A span $\mathbb{R}^m$. **(06 Marks)**

c. For $T(x_1, x_2) = (3x_1 + x_2, 5x_1 + 7x_2, x_1 + 3x_2)$. Show that T is a one to one linear transformation. Does T maps $\mathbb{R}^2$ onto $\mathbb{R}^3$. **(07 Marks)**

8 a. Let W = span $\{x_1, x_2\}$ where $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$x. Construct an orthonormal basis $\{v_1, v_2\}$ for w. **(10 Marks)**

b. Find a least-squares solution of the inconsistent system

$$Ax = b \text{ for } A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \; b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$$ **(10 Marks)**

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